

Fig. 3 Comparison of predicted and measured pitot pressure for  $\text{CO}_2$  test gas.

measured flow quantities to agree best with equilibrium prediction accounting for flow attenuation and including a totally reflected shock at the secondary diaphragm. No flow attenuation was observed for the helium tests.

In conclusion, the present results show the existence of a uniform test core diameter of approximately half the expansion tube diameter for helium, argon, air, and  $\text{CO}_2$  test gases. This test core diameter and duration of test flow are sufficient for model testing with current high response, miniaturized instrumentation techniques. Comparison of predicted and measured flow quantities suggest the expansion to be near thermochemical equilibrium for all gases and implies the existence of a totally reflected shock at the secondary diaphragm. Argon, air, and  $\text{CO}_2$  flows were observed to attenuate while traversing the acceleration section, whereas no attenuation was observed for helium.

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## Experimental Investigation of Gas Injection through a Transverse Slot into a Subsonic Cross Flow

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#### Introduction

EFFECTIVE fuel injectors for combustion chambers must produce rapid mixing of the fuel with the mainstream. Lateral injection offers the advantage of adding the effect of inertial penetration to turbulent diffusion. Penetration of laterally injected jets is of interest also for other situations, such as cooling of gas turbines or boundary-layer control.

In the experiments reported here, both the airstream and the jet were subsonic with the jet being injected through a long and narrow, transverse slot. Results of these experiments and comparison with other data are presented in the following. More details on the experimental techniques are described elsewhere.<sup>1</sup>

Many experiments on jet penetration in a cross flow have been reported in the literature.<sup>2-15</sup> Jets with various shapes of their cross section have been used, but the ratio of the longest to the shortest dimension (aspect ratio) did not exceed about four. An exception is the study by Vranos and Nolan<sup>2</sup> which included injection of helium through a circumferential slot (infinite aspect ratio) into supersonic flow. There is a lack of data on jet penetration for slots with aspect ratios considerably larger than four.

#### Experiments and Results

In the present experiments, the slot width was 0.01 in. (0.25 mm) and the length was 2.5 in. (64.5 mm), corresponding to an aspect ratio of 250. Air was supplied by a centrifugal blower through a plenum chamber into a square duct and test section of 4.25 in. (108 mm) width, and its velocity ranged from 26 to 259 fps (7.9-79 m/sec). The injected gases were helium, Freon 22, and Freon 116. The ratio of the jet density to the air density thus varied between about 0.14 and 4.8. Jet velocities ranged from 117 to 1820 fps (36-556 m/sec). The parallel section of the slot passage was 0.125 in. (3.2 mm) long, and the flow through it was supplied through a rounded inlet from a small plenum chamber to which the gas was fed from compressed-gas bottles through controls and flowmeters.

Photographs of the jets were obtained with a standard single-pass schlieren system, and Fig. 1 shows three photographs of a helium jet injected into different airflows. The reduced jet penetration with increased stream velocity and the tendency to early reattachment to the wall on the side of the injection slot are clearly noticeable. The black step in the lower right corner of each picture is a marker attached to the window of the test section to provide the reference scale needed for evaluation of the photographs in actual dimensions.

Penetration was defined by the leading edge of the jet visible in the photographs and was measured at 2.06 and 4.14 in. (52 and 105 mm) from the injection point. These measurements were correlated in terms of the momentum-flux ratio  $J =$

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Index categories: Jets, Wakes, and Viscid-Inviscid Flow Interactions; Nozzle and Channel Flow; Subsonic and Transonic Flow.

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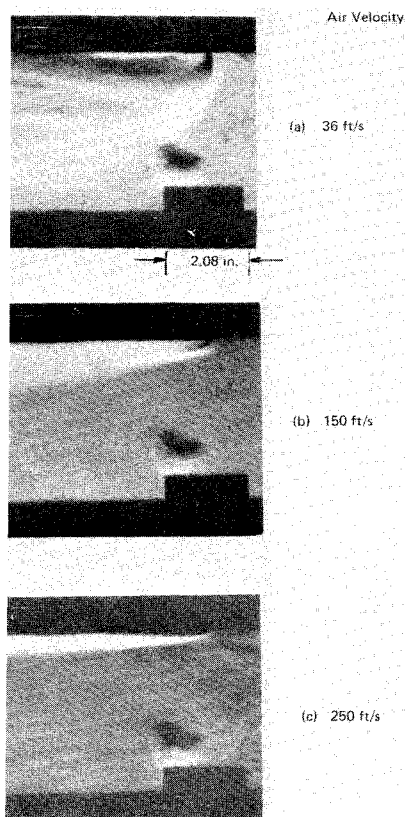


Fig. 1 Schlieren photographs of a helium jet (1470 fps) into air flows. (The injection slot is 0.01 in. wide and 2.5 in. long.)

$(\rho_j V_j^2)/(\rho_s V_s^2)$ , where  $\rho$  and  $V$  are density and velocity and subscripts  $j$  and  $s$  refer to the jet and to the airstream. The ratio  $J$  ranged from about 4 to 700, and all data are collected in Fig. 2, where the dimensionless penetration  $y/w$  ( $w$  is the slot width) is plotted vs  $J$  for the two distances  $x$  from the injection point where the measurements were made. To maintain clarity of the plot, the results for the two values of  $x$  are offset by one decade of the logarithmic scale.

It can be seen that the data are well correlated by two parallel straight lines. Few data points fall more than 15% above or below the correlation line, and most are considerably closer. The correlation lines are best expressed as

$$\frac{y}{w} = 3.24 J^{0.48} \left( \frac{x}{w} \right)^{0.34} \quad (1)$$

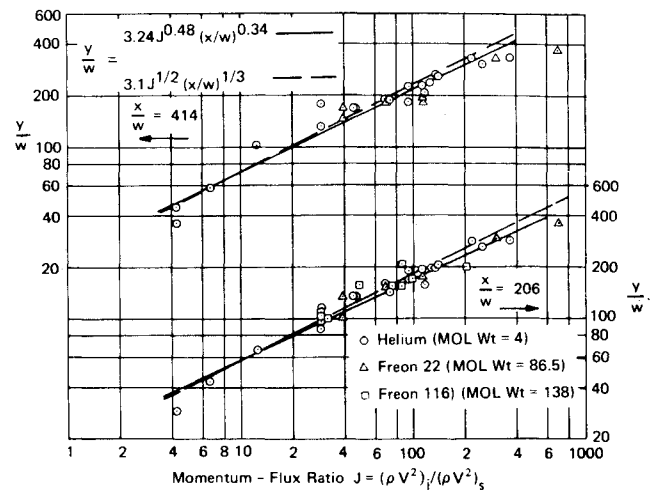


Fig. 2 Penetration of gas jets injected into subsonic cross flow through a narrow slot.

A more convenient correlation than Eq. (1) can be obtained by using simpler exponents and making a slight adjustment of the constant factor. The relationship

$$\frac{y}{w} = 3.1 J^{1/2} \left( \frac{x}{w} \right)^{1/3} \quad (2)$$

is entered in Fig. 2 as broken lines. This simplified correlation appears entirely adequate for all practical purposes.

#### Discussion

Results for jet penetration often are presented in the form of Eqs. (1) or (2), and data from several publications are collected in Table 1, where  $A$ ,  $B$ , and  $C$  represent the constant factor and the exponents of  $J$  and  $x/w$ . The exponent of  $J$  has generally been found to be near 0.5. As indicated in Table 1, investigations have dealt with various shapes of the injected jets. Measurements have been based on schlieren photography, visualization by smoke, pitot-tubes, and probes that measure temperature or concentration of the jet gas. Jet penetration has been defined as the location of the centerline, the visible edge of the jet, or a selected temperature or concentration of the jet gas near the edge of the jet. Considering the great variety of test conditions and techniques, the results in Table 1 are rather consistent. The large differences in the constant factor of the correlations,  $A$ , probably are due primarily to differences in the definition of penetration. It must also be kept in mind that comparison of the constants in Table 1 beyond their experimental range may be questionable.

Table 1 Comparison of correlation data for jet penetration  
[ $y/w = AJ^B(x/w)^C$ ]

Investigators	Ref.	Jet	Penetration	$(x/w)_{\max}$	$A$	$B$	$C$
<i>a</i>							
Present Study		Long slot	Edge	400	3.24	0.48	0.34
					(3.1)	(0.5)	(0.333)
Vranos & Nolan <sup>b</sup>	(2)	Circumf. slot	Edge		2.33	0.642	0.413
Vranos & Nolan <sup>b</sup>		Circular	Edge	72	1.56	0.47	0.0866
Vranos & Nolan <sup>b</sup>		Elliptic (4:1)	Edge		3.54	<sup>c</sup>	0.0694
Callaghan & Ruggeri	(3)	Circular	Edge	150	1.91	<sup>d</sup>	0.303
Shandorov	(6)	Circular	Centerline	5	1.0	0.39	0.39
Ivanov	(5)	Circular	Centerline	5	1.0	0.43	0.333
Kamotani & Greber	(11)	Circular	Centerline	20	0.89	0.47	0.36
Keffer & Baines <sup>e</sup>	(7)	Circular	Centerline	5	0.51	0.5	0.5
Hurn & Akers	(15)	Circular	Centerline				0.333

<sup>a</sup> Reference length  $w$  is slot width, diameter of circular jet, diameter of equivalent circle (equal area) for elliptic jets.

<sup>b</sup> Supersonic flow.

<sup>c</sup> Correlation based on  $(\rho_j/\rho_s)^{0.466} (V_j/V_s)^{1.294}$ .

<sup>d</sup> Correlation based on  $[(\rho_j V_j)/(\rho_s V_s)]^{0.606}$ .

<sup>e</sup> Constant density ratio;  $A$ ,  $B$ , and  $C$  estimated from Fig. 4 of the reference. Penetration is still increasing at  $x/w = 100$ .

A large difference between the jet and stream densities raises the question whether or not buoyancy could have introduced errors into the measurements, but a simple estimate<sup>1</sup> showed that this error is insignificant for the present experiments.

The present results have been obtained for a single slot width. The experiments by Vranos and Nolan<sup>2</sup> with circumferential slots (infinite aspect ratio) varying in width between 0.045 and 0.14 in. did not indicate any influence of the slot width on their correlation. For elliptic jets of small aspect ratio (up to 4), they found that the best reference value for  $w$  is the diameter of a circle having the same cross-sectional area as the jet (rather than the hydraulic diameter). The correlation thus depends on the aspect ratio of the jet. For circular jets, they observed no effect of jet diameter when the latter was reduced by factors of 1.4 and 2. Ivanov<sup>5</sup> and Shandorov<sup>6</sup> also found no effect of jet size for circular jets when the diameter was increased by about 50%. Similarly, Callaghan, Ruggeri, and Bowden<sup>4</sup> found no effect of jet size for elliptic jets (aspect ratio of 4) when the equivalent jet diameter was reduced by a factor 0.6. Therefore, Eq. (1) or (2) should be satisfactory for some range of the slot width as long as the aspect ratio remains sufficiently large. For the correlation of penetration data for narrow slots (high aspect ratio), the slot width appears to be the appropriate reference dimension. Since the best reference dimension for jets with small aspect ratios is the diameter of a circle having the same cross-sectional area, the appropriate reference dimension must be a function of the jet aspect ratio. Because of its practical importance, this relationship should be sought in future work.

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## Comparison of Calculated Static and Dynamic Collapse Pressures for Clamped Spherical Domes

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#### Introduction

THIS Note compares the results of static and dynamic analyses of steel pressure domes used in certain applications at Lawrence Livermore Lab. Figure 1 shows a typical spherical dome that has been built into a thick pipe; the thickness of the dome varies, but a uniform surface loading is applied on the convex surface. The mode of failure of such domes is by sudden collapse—a highly nonlinear problem involving large deformation, large displacements, and elastic-plastic flow.

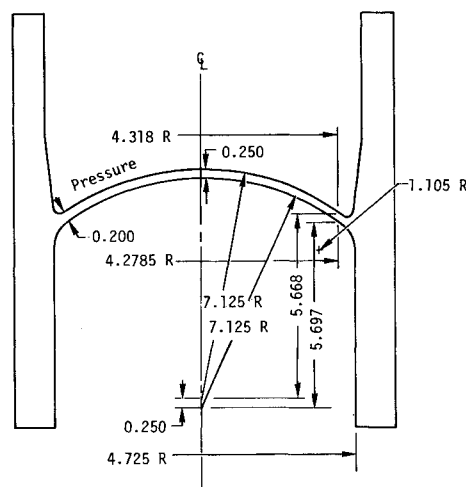


Fig. 1 Dome with pressure on the convex side. All dimensions are in inches.

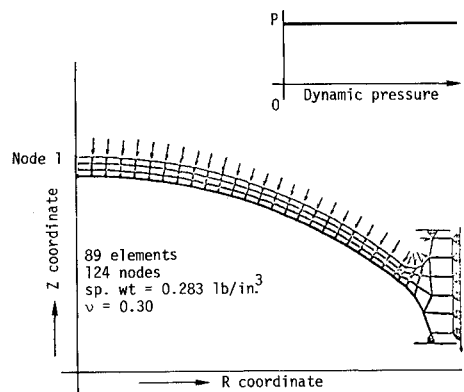


Fig. 2 Finite element model.

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Index categories: Structural Dynamic Analysis; Structural Stability Analysis; Structural Static Analysis.

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